# Differential-Turning Optimality Criteria

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**Circling encounters between aircraft modeled in energy approximation are studied in a differential game setting. Capture is defined in terms of turn-angle and a derived altitude-match requirement. Target sets and their capture subsets are defined and the computation of boundaries discussed and illustrated. A sufficient condition guaranteeing capture of an optimally-evading opponent in a turning chase is obtained which involves energy-rate/turn-rate "hodograph figure" comparison.**

## Introduction

STANDARD practice for maneuvering performance comparison of conventional fighter aircraft is to overlay contour plots of excess power and sustainable turn-rate in the Mach-altitude chart. For the type of protracted turning competition that spirals down to low altitude and subsonic/transonic speeds, a rough indication of superiority is furnished by comparison of the maximum sustainable turn rates for the two craft, which are ordinarily developed in this region. Although superiority in maximum sustainable turn rate permits a pursuer to close the gap in a turning chase, actually realizing a firing opportunity depends as well upon his closing off an upward escape route as the attack is pressed. This ability to stay with an evader in a pull-up requires adequate specific energy in the final phase of closure, and the requirement has important implications on the ability of the pursuer to develop a closing turn-rate in this phase. The effect will be marked if the two opposing craft exhibit their best sustainable turn-rates at substantially different velocities, and the likelihood of this is increased if one has variable geometry or thrust-augmentation of lift, or if both have such features in differing fashions and amounts. Turn-rate comparison criteria suitable for such situations are presented in the following.

The analytical approach taken makes use of asymptotic ("energy") modeling[1,2] and a differential gaming formulation similar to that of the earlier study of Ref. 3. The present analysis drops the requirement of altitude proximity between pursuer and evader considered earlier, except for matching of the altitudes at the terminal point.

## Analytical Formulation

Consider as a preliminary the problem of a single aircraft turning through maximum heading change in fixed time with final altitude and path angle to the horizontal specified, final energy open. The equations of motion are

$$\dot{h} = V \sin \gamma \tag{1}$$

$$\dot{\gamma} = \frac{g}{V}\left[\frac{L \cos\mu}{W} - \cos\gamma\right] \tag{2}$$

$$\dot{E} = \frac{(T-D)V}{W} \tag{3}$$

$$\dot{\chi} = \frac{gL \sin \mu}{VW} \tag{4}$$

These contain a thrust-along-the-path simplification which, however, is not essential. Here $E = [h + (V^2/2g)]$ is specific energy, $\chi$ is heading angle, and $\mu$ is bank angle.

When an order reduction to an energy model is effected, the 'lost' boundary conditions are the two on altitude and path angle, this shortcoming of the reduced-order approximation to be alleviated with a "boundary-layer" correction.[2] The order-reduction yields the relationships $\sin \gamma = 0$ and

$$L \cos\mu = W \cos \gamma \tag{5}$$

These determine path angle $\gamma$ and lift coefficient $C_L$.

$$\gamma = 0 \tag{6}$$

$$C_L = \frac{W}{qS \cos\mu} \tag{7}$$

The inequality constraint on lift coefficient

$$C_L \leq \bar{C}_L(M) \tag{8}$$

translates into

$$\frac{W}{qS \cos\mu} \leq \bar{C}_L(M) \tag{9}$$

which, for purposes of the reduced-order solution, is an inequality constraint involving both state and control variables. The highest altitude allowed by this constraint, which occurs for $\mu = 0$, must, at the terminal point of the path, equal or exceed the specified final altitude. Evaluated at the specified altitude, the constraint thus becomes a terminal state inequality restricting the final specific energy.

At each energy level, there is an upper bound on altitude imposed by vertical equilibrium and maximum lift coefficient

$$h_L(E) - h \geq 0 \tag{10}$$

which might be termed the aircraft's "loft-ceiling." This loft-ceiling of a VTO will, at low energy, equal the specific energy since hovering at zero velocity is possible, while for a conventional aircraft, maximum lift coefficient will be the main determining factor. Between these extremes, the vertical component of thrust and/or augmentation of lift will be important in varying degrees.

## Reduced-Order Capture

The reduced-order system of differential equations for each vehicle has the form of the pair (3) and (4) with $E$

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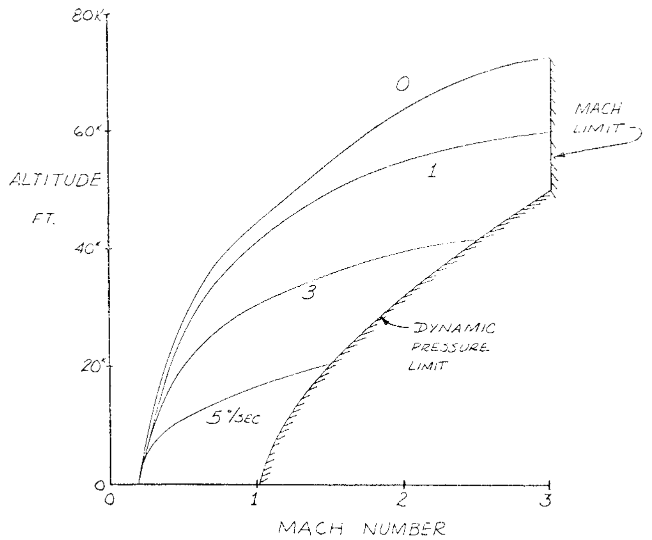


Fig. 1 Sustainable turn rates—Aircraft A.

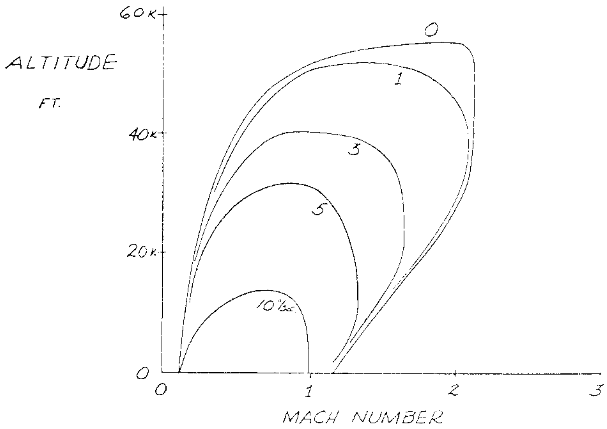


Fig. 2 Sustainable turn rates—Aircraft B.

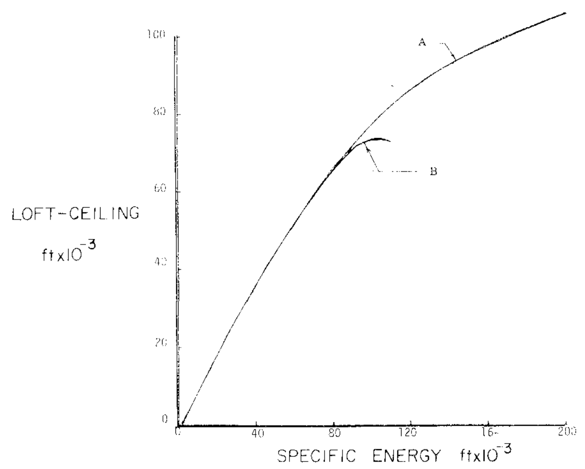


Fig. 3 Loft-ceiling vs energy.

and $\chi$ as state and bank angle $\mu$, altitude h, and throttle $\eta$ (an argument of $T$) as controls. Altitude match may be treated in terms of a specification that the pursuer's specific energy be high enough that his upper altitude bound equal or exceed the evader's. In this conception, capture would consist of angular closure, i.e., heading-match, plus a requirement that the pursuer's loft-ceiling equal or exceed the evader's.

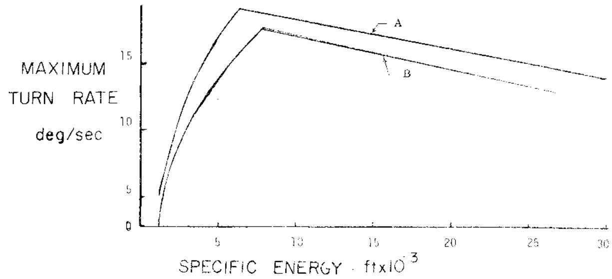


Fig. 4 Maximum turn rate vs energy.

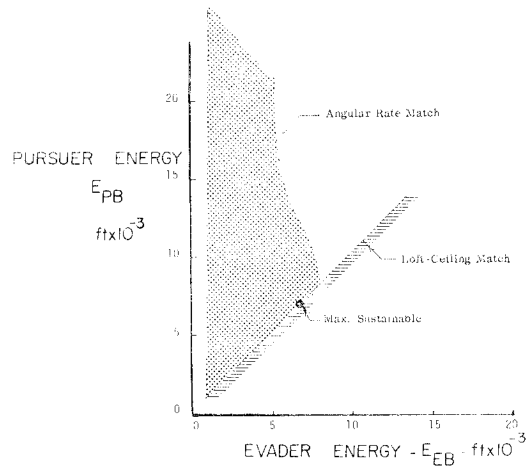


Fig. 5 Pursuer/evader B/B—target and capture sets.

## Target and Capture Sets

Data for two aircraft previously examined for turning performance are used for illustration. Flight envelopes and sustainable turn-rates from Ref. 3 are shown in Figs. 1 and 2. Aircraft A is a hypothetical Mach 3 aircraft whose data are given in Ref. 4. Aircraft B is a version of the F-4. Loft-ceiling vs specific energy is shown for both craft in Fig. 3; the close proximity of the two curves at low energies is coincidental. Maximum turn rates for both aircraft are shown vs energy in Fig. 4; the intersections of the aerodynamic and structural limits occur at the energy corresponding to the 'corner velocity' at sea level.[5]

The *target set* for an aircraft of type B pursuing a similar craft is shown in Fig. 5; it is that portion of the state subspace $\Delta\chi = 0$ for which the pursuer's loft-ceiling equals or exceeds the evader's, i.e., the portion on and above the curve labeled "Loft-Ceiling Match." The shaded portion of the target set is the region in which the pursuer's maximum turn-rate exceeds the evader's, as determined by the characteristics of Fig. 4; this will be called the *capture set*. The remaining portion of the target set will not see capture except at time zero for initial conditions corresponding to capture.

Target and capture sets for three combinations of aircraft are shown in Figs. 5–7. Figure 5 is for aircraft of type B in rôles of both pursuer and evader. Figure 6 is for A chasing B, and Fig. 7 for B chasing A. It should be noted that the possibility of capture at high evader energies depends upon superiority of the pursuer in maximum lift and/or structural limit load factor.

## A Sufficient Condition

A comparison of turn-rates and energy-rates of pursuer and evader carried out at energies for equal loft-ceilings leads to a condition whose satisfaction guarantees cap-

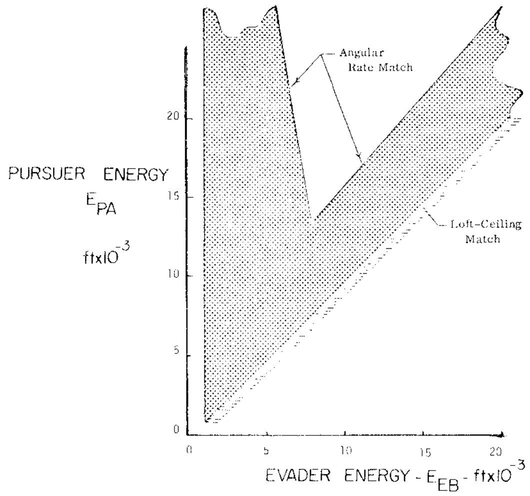


**Fig. 6 Pursuer/evader A/B—target and capture sets.**

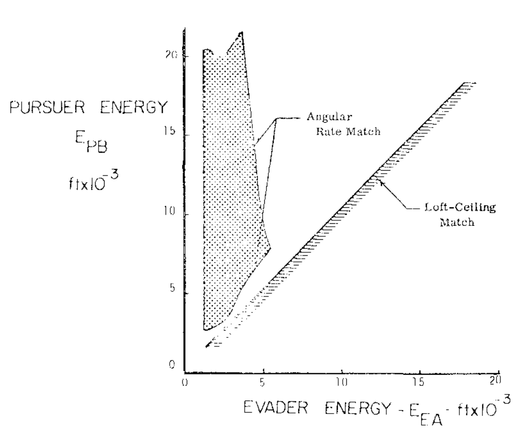


**Fig. 7 Pursuer/evader B/A—target and capture sets.**

ture. Elementary arguments are adequate to demonstrate this because the condition applies only to *eventual* capture and does not require minimax-time-optimal maneuvers even as references.

Consider the domains of maneuverability of the two craft in the $(\dot{\chi},\dot{h}_L)$ space, the evader's superimposed on the pursuer's, with energies of the two chosen so that the loft-ceilings are equal (Fig. 8). These figures, nonconvex in the setting of the original problem, are assumed to have been made convex by "relaxation" in the usual way for hodograph figures.[2,6,7]

If the pursuer's figure completely envelops the evader's, without contact, for all loft-ceilings attainable by the evader, capture is guaranteed. To see this, imagine that the pursuer first matches loft-ceiling with the evader, which he can do by virtue of the assumed superior $\dot{E}$ capability; after this he follows in loft-ceiling, using the turn-rate margin implicit in the envelopment of one figure by the other to close angularly. Because the loft-ceiling constraint is an inequality, it seems at least a possibility to obtain a less restrictive sufficient condition, one relinquishing energy-rate margin in the left portion of the figure, say, with more intricate arguments.

The comparison of superposed hodograph figures might be thought of as a generalization of the turn-rate/energy-rate trade-off study technique of Ref. 8 to a pursuit/evasion setting.

Sustainable turn-rates and level-flight energy-rates are plotted vs loft-ceiling in Figs. 9 and 10 for both aircraft. The sustainable turn-rate associated with each loft-ceiling value is developed at an altitude determined by scanning the altitude range for the maximum sustainable

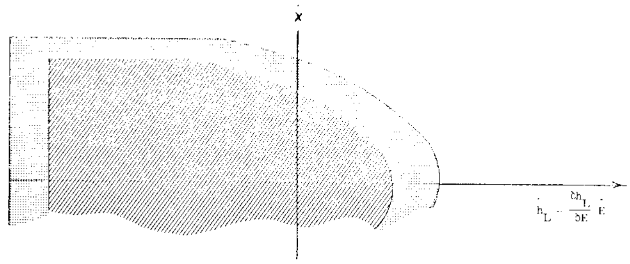


**Fig. 8 Superposition of hodograph figures at equal loft-ceilings.**

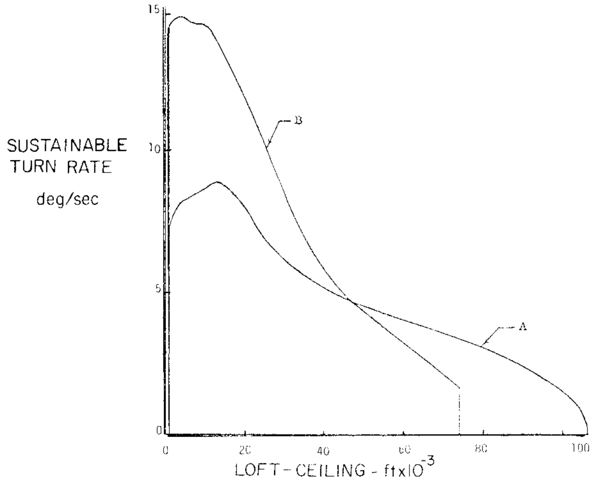


**Fig. 9 Sustainable turn rate vs loft-ceiling.**

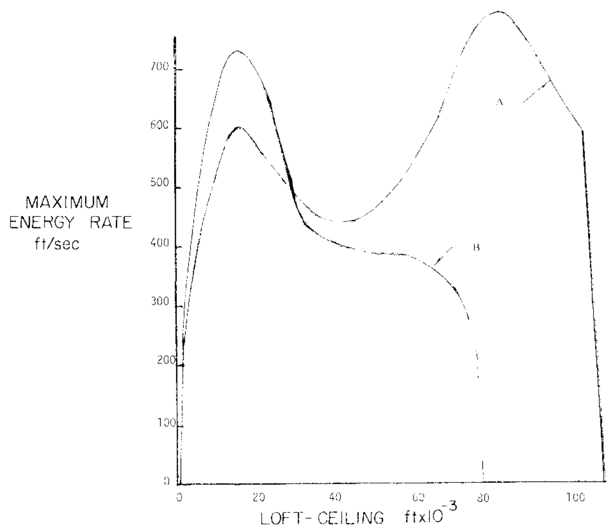


**Fig. 10 Maximum energy rate vs loft-ceiling.**

turn-rate at the particular energy. The level-flight energy-rate is similarly determined by a scan. The sufficient condition is not satisfied with either $A$ pursuing $B$ or $B$ pursuing $A$.

## Conclusions

The target and capture set computations and the hodograph comparison at matched loft-ceiling appear to be of value for comparing fighter aircraft designs and perhaps as criteria for designing against "threat" aircraft configurations. Of course, any over-all comparison of designs should include dash performance and, in this connection, a "differential-dash" analysis analogous to the present one appears to be of interest; one would compare sustainable speeds (i.e., flight-envelope speeds) at matched loft-ceilings, etc. in a study of a 2-D vertical-plane chase. An account of differential-turn-maneuvering tactics is given in Ref. 9.

### References

[1]Kelley, H. J. and Lefton, L., "Supersonic Aircraft Energy Turns," Fifth IFAC Congress, Paris, France, 1972; also *Automatica*, Vol. 8, Sept. 1972, pp. 575–580.

[2]Kelley, H. J., "Aircraft Maneuver Optimization by Reduced-Order Approximation," in Vol. X of *Control and Dynamic Systems: Advances in Theory and Applications*, edited by C. T. Leondes, Academic Press, New York, 1973.

[3]Kelley, H. J. and Lefton, L., "Differential Turns," *AIAA Journal*, Vol. 11, No. 6, June 1973, pp. 858–861.

[4]Falco, M. and Kelley, H. J., "Aircraft Symmetric Flight Optimization," in Vol. X of *Control and Dynamic Systems: Advances in Theory and Applications*, edited by C. T. Leondes, Academic Press, New York, 1973.

[5]Preyss, A. E. and Willes, R. E., "Air Combat Maneuvering," USAF Academy Notes, 1969; also Vol. II of *Advanced Aircraft Propulsion/Engagement Study*, TR 71-7, USAF Academy, Colorado Springs, Colo., Sept. 1971.

[6]Kelley, H. J. and Sullivan, H. C., "Roll-Modulated Lifting Entry Optimization," *AIAA Journal*, Vol. 11, No. 7, July 1973, pp. 913–915.

[7]Contensou, P., "Etude Théorique des Trajectoires dans un Champ de Gravitation. Application au Cas d'un Centre d'Attraction Unique," *Astronautica Acta*, Vol. VIII, Fasc. 2 3, 1962, pp. 134–150.

[8]Boyd, J. R., "Maximum Maneuver Concept," informal briefing, USAF Systems Command Headquarters, Andrews Air Force Base, Md., July 1971; also briefing notes by J. R. Boyd, T. P. Christie, and R. E. Drabant, Eglin Air Force Base, Fla., Aug. 1971.

[9]Kelley, H. J., "Differential-Turning Tactics," Paper 74-815, AIAA/AAS Mechanics and Control of Flight Conference, Anaheim, Calif., 1974.

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# Coordinated Adaptive Washout for Motion Simulators

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**This paper introduces a new method of providing motion cues to a moving base six-degree-of-freedom flight simulator utilizing nonlinear filters. Coordinated adaptive filters, used to coordinate translational and rotational motion, are derived based on the method of continuous steepest descent, and the basic concept of the digital controllers used for the uncoordinated heave and yaw cues is also presented. The coordinated adaptive washout method is illustrated by an application in a six-degree-of-freedom fixed-base environment.**

## Nomenclature

| | |
|---|---|
| $A$ | = angular position break point, rad |
| $a_x, a_y, a_z$ | = aircraft body axis translational accelerations, m/sec$^2$ |
| $B$ | = angular velocity threshold, rad/sec |
| $b_x, b_y$ | = coefficient for position penalty in cost function, per sec$^4$ |
| $C$ | = angular velocity washout rate, rad/sec$^2$ |
| $c_x, c_y$ | = coefficient for velocity penalty in cost function, per sec$^2$ |
| $d_x, d_y$ | = damping parameters for second-order translational washout, rad/sec |
| $e_x, e_y$ | = frequency parameters for second-order translational washout, rad/sec$^2$ |
| $f_{i,x};f_{i,y};f_{i,z}$ | = inertial axis translational acceleration commands, m/sec$^2$ |
| $f_{s,x}, f_{s,y}$ | = body axis longitudinal and lateral acceleration at motion simulator centroid location, m/sec$^2$ |
| $f_{s,z}$ | = body axis vertical acceleration (referenced about 1g) at motion simulator centroid location, m/sec$^2$ |
| $g$ | = gravitational constant, m/sec$^2$ |
| $J_x, J_y$ | = longitudinal and lateral cost function |
| $K_D$ | = digital controller gain parameter |
| $K_{x,1}; K_{x,2}; K_{x,3}$ | = longitudinal gain parameters |
| $K_{y,1}; K_{y,2}; K_{y,3}$ | = lateral gain parameters |
| $p,q,r$ | = body axis angular velocity commands, rad/sec |
| $p_a, q_a, r_a$ | = body axis aircraft angular velocities, rad/sec |
| $p_{x,1}; p_{x,2}; p_{x,3}$ | = longitudinal adaptive parameters |
| $p_{y,1}; p_{y,2}; p_{y,3}$ | = lateral adaptive parameters |
| $t$ | = time, sec |
| $t_1, t_2$ | = arbitrary time, sec |
| $W_x, W_y$ | = angular rate weighting coefficient, m$^4$/rad$^2$sec$^2$ |
| $x,y,z$ | = commanded inertial translational position of motion simulator, m |
| $\psi,\theta,\phi$ | = commanded inertial angular position of motion simulator, rad |
| $\dot{\psi}_a, \dot{\theta}_a, \dot{\phi}_a$ | = angular velocity input commands, rad/sec‡ |
| $\tau_1, \tau_2$ | = time when $\psi$ reaches breakpoint, $A$, sec |

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## I. Introduction

A NEW method of providing motion cues to a moving base six-degree-of-freedom flight simulation has been developed at Langley Research Center. The method, coordinated adaptive washout, is based on the idea of coordination of rotation and translation to obtain accurate longitudinal and lateral force cues in a manner similar to the work of Schmidt and Conrad.[1,2] The major differences between the subject scheme and the work of Schmidt and

‡A dot over a variable indicates the time derivitive of that variable.